The geometry of our space would then be like that of a tridimensional sphere in a four dimensional manifoldness. This representation carries with it that the angle-sum in a triangle, as in ordinary spherical triangles, is greater than two right angles, and indeed the more so, the greater the triangle The straight would then have no point at infinity, and through a given point no parallel to a given straight could be drawn. Now Cayley constructed his celebrated projective metrics to show how the ordinary Euclidean metrics may be taken as a special part of projective geometry. Klein generalizes Cayley and founds three metric geometries, the elliptic (Riemann's), the hyperbolic (Lobachevsky's), the parabolic,(Euclid's).

This little paper of 1871 contains the promise of much that is most genial in the after work of a man now generally considered as the most interesting and one of the very greatest of living mathematicians. Of all those splended and charming series of lectures with which Klein has made Goettingen so attractive to the whole world, the most delightful and epoch-making are those on non-Euclidean geometry, (Nicht-Euklidische Geometrie, I. Vorlesung,gehalten waehrend des Wintersemesters 1889-90 von F. Klein. Ausgearbeitet von Fr. Schilling. Zweiter Abdruck. Goettingen 1893. Small Quarto, lithographed, pp. v. 365. II. Sommersemesters 1890. Zweiter Abdruck 1893. pp. iv. 238.)

The World's Science Congress at Chicago was in nothing more fortunate than in the presence of Helmholtz and Felix Klein, and in the spontaneous and universal homage accorded them no idea was more often emphasized than their connection with the birth and development of that wonderful new world of pure science typified in the non-Euclidean geometry.

The narrow limits of this feeble sketch prevent the statement of how much promise, richly fulfilled in the development of this many-sided man, in totally other directions is contained in a little-known paper of 1873, "Ueber den allgemeinen Functionsbegriff und dessen Darstellung durch eine willkuerliche Curve."

Twenty years of production and achievement have not in the least dampened the ardour of this enthusiastic mind. This very summer at the great meeting of scientists in Vienna Klein seemed the busyest, the foremost of all that goodly company.

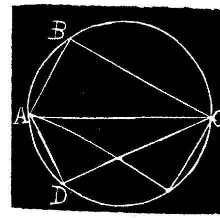
ISOPERIMETRY WITHOUT CURVES OR CALCULUS.

By Professor P. H. PHILBRICK, M. Sc., C. E., Lake Charles, Louisiana.

[Continued from the November Number.]

PROPOSITION VII. If in a quadrilateral two sides are equal, and the

other two sides are also equal; the quadrilateral may be inscribed either with the equal sides adjacent giving $ABCD$, or with the equal sides opposite giving $ABCD$.

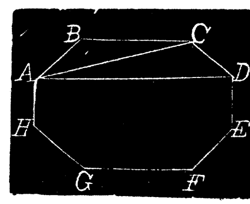


For AC is a diameter of the circumscribing circle in either case.

PROPOSITION VII. To find the polygon of n sides of given perimeter and maximum area.

Let $ABCDEF$ etc. represent the polygon.

Draw the diagonal AC, and suppose the sides AB and BC to vary, while the other sides remain fixed in magnitude and position, and we prove, as in Proposition II, that the area of ABC, and consequently of the polygon is a maximum, when $AB=BC$. Similarly $BC=CD$, $CD=DE$, etc.



Therefore the polygon is equilateral.

Draw the diagonal AD. Then supposing AB, BC and CD to vary in *position*, while the other sides remain entirely fixed, we prove, as in Proposition V, that the angles at B and C are equal. We prove in the same way that the angles are equal at C and D, at D and E, etc; and hence the angles are all equal.

The polygon is therefore both equilateral and equiangular (that is regular) and is inscriptable in a circle.

PROPOSITION VIII. Of all Polygons having the same area and the same number of sides, the regular polygon has the maximum perimeter.

For let P be a regular polygon and M any irregular polygon, having the same number of sides and area.

Now if the perimeter of P was equal to that of M then (Prop. VII) the area of P would be greater than that of M, but since it is equal to that of M its perimeter must be less.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton), Ph. D., (Johns Hopkins), Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the November Number.]

PROPOSITION XII. *Again I say the straight AD somewhere on that side will meet the straight PL (and indeed at a finite, or terminated distance) also in the hypothesis of obtuse angle.*